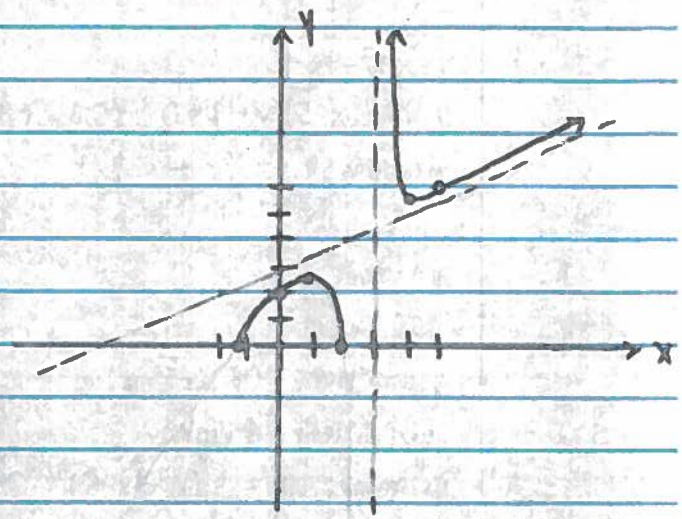
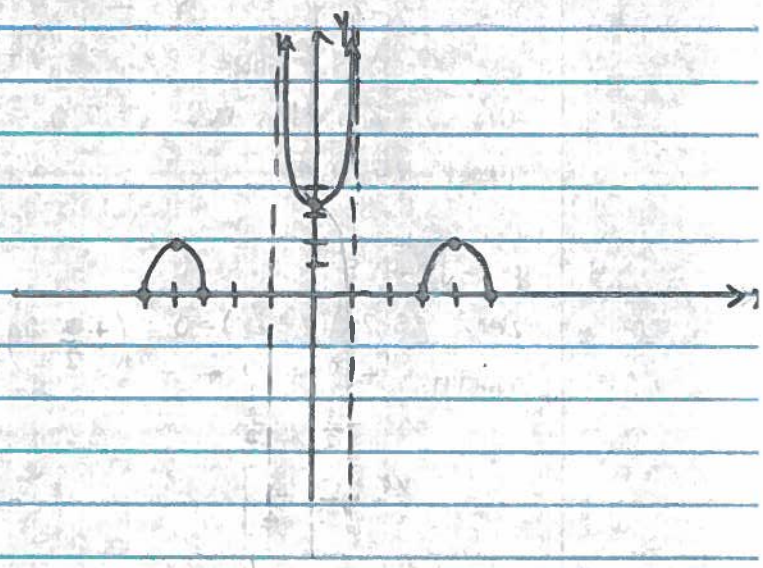
$\boxed{5\text{-}4}$ Homework: p. 298, #1–12

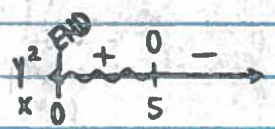
1. domain: $x \in [-1.2, 2] \cup (3, 5), (5, \infty)$
   VA: $x = 3$
   zeros: $(-1.2, 0), (2, 0)$
   y-int: $(0, 2)$
   extremes: $(-1.2, 0), (1, 2.5), (2, 0), (4, 5.8)$
   EB: left none; right $y = \frac{1}{2}x + 3$
   POE: $(5, 6)$
   range: $y \in [0, 2.5] \cup [5.8, \infty)$

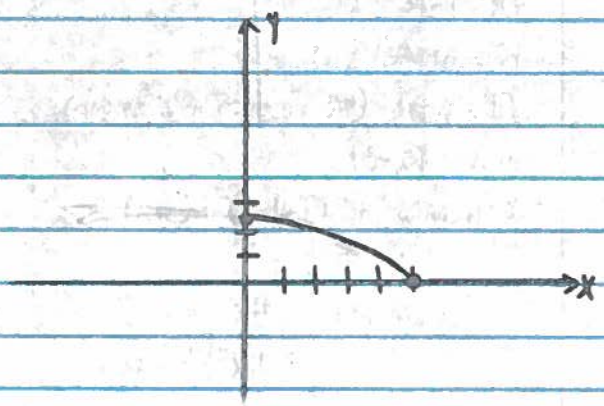


2. domain: $x \in [-5, -3] \cup (-1, 1) \cup [3, 5]$
   zeros: $(-5, 0), (-3, 0), (3, 0), (5, 0)$
   VA: $x = \pm 1$
   POE: none
   y-int: $(0, 3.5)$
   extremes: $(-5, 0), (-4, 2), (-3, 0), (0, 3.5),$
   $(3, 0), (4, 2), (5, 0)$
   EB: left none; right none
   range: $y \in [0, 2] \cup [3.5, \infty)$

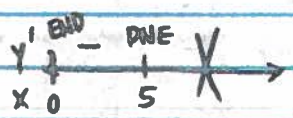


6. $y = \sqrt{5 - x}$ on $x \in [0, 5]$
   zeros: $(5, 0)$
   domain: $x \in [0, 5]$
   VA: none
   POE: none
   y-int: $(0, \sqrt{5}) \approx (0, 2.236)$
   EB: none b/c domain
   extremes: $y' = \frac{1}{2}(5 - x)^{-1/2}(-1)$





$$y' = \frac{-1}{2(5-x)^{1/2}} = 0 \quad \text{DNE}$$

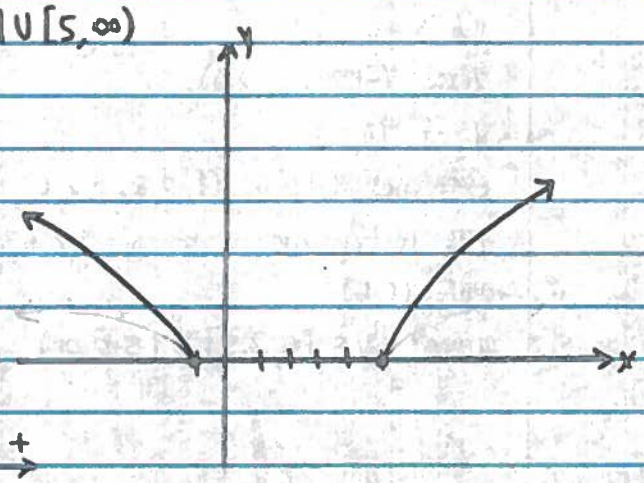


cv: $x = 5$; ev: $y = 0 \therefore (5, 0)$
$x = 0$; $y = \sqrt{5} \therefore (0, \sqrt{5})$
range: $y \in [0, \sqrt{5}]$

3. $y=\sqrt{x^2-4x-5}$

zeros: $(x-5)(x+1)=0$; $(5,0),(-1,0)$

domain: y^2 $+$ 0 $-$ 0 $+$; $x\in(-\infty,-1]\cup[5,\infty)$
x -1 5



VA: none

POE: none

y-int: none (b/c $\sqrt{-5}=i$)

EB: ↑ left ; ↑ right

extremes: $y'=\frac{1}{\cancel{2}}(x^2-4x-5)^{-1/2}(\overset{(x-2)}{\cancel{2x-4}})$

$y'=\dfrac{x-2=0}{(x^2-4x-5)^{1/2}}$ DNE

y' $-$ DNE 0 DNE $+$
x -1 2 5

cv's: $x=\cancel{2},5,-1$; cv's: $y=0,0$ ∴ $(5,0),(-1,0)$

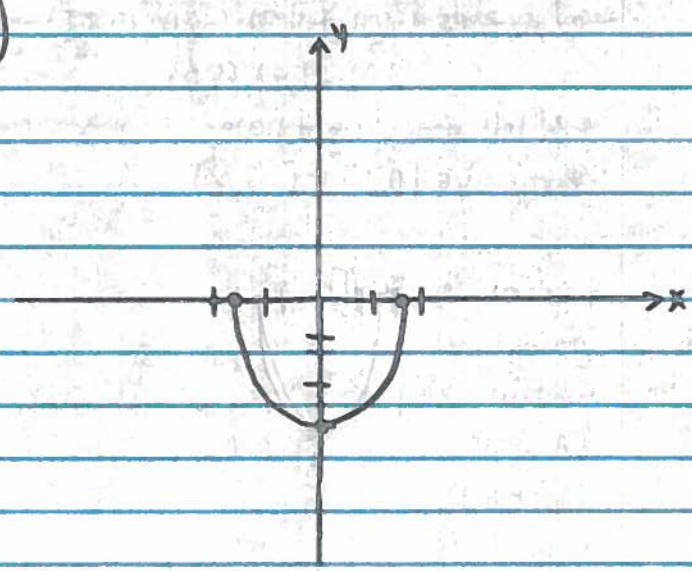
range: $y\in[0,\infty)$

4. $y=-\sqrt{9-4x^2}$

zeros: $(3-2x)(3+2x)=0$; $\left(\pm\frac{3}{2},0\right)$

domain: y^2 $-$ 0 $+$ 0 $-$
x $-\frac{3}{2}$ $\frac{3}{2}$

$x\in\left[\frac{-3}{2},\frac{3}{2}\right]$



VA: none

POE: none

EB: none (b/c no $\pm\infty$ in domain)

y-int: $(0,-3)$

extremes: $\frac{+1}{\cancel{2}}(9-4x^2)^{-1/2}(\overset{4x}{\cancel{+8x}})$

$y'=\dfrac{4x=0}{(9-4x^2)^{1/2}=\text{DNE}}$

y' DNE $-$ 0 $+$ DNE
x $-\frac{3}{2}$ 0 $\frac{3}{2}$

cv's: $x=0,\pm\frac{3}{2}$ ∴ $(0,-3),\left(\pm\frac{3}{2},0\right)$

range: $y\in[-3,0]$

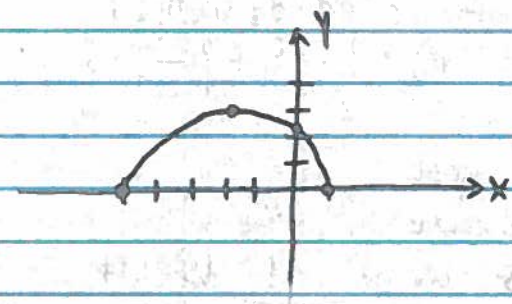
5. $y=\sqrt{5-4x-x^2}$

zeros: $-(x^2+4x-5)=0$

$(x+5)(x-1)=0$

$(-5,0),(1,0)$

domain: y^2: $-$ 0 $+$ 0 $-$; x: -5, 1 $\quad x\in[-5,1]$



VA: none

POE: none

EB: none b/c domain

y-int: $(0,\sqrt{5})\approx(0,2.236)$

extremes: $y'=\frac{1}{2}(5-4x-x^2)^{-1/2}(-4-2x)$

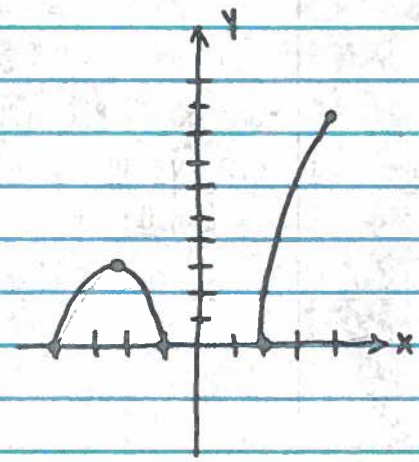
$$=\frac{-\cancel{2}(x+2)}{\cancel{2}(5-4x-x^2)^{1/2}}=0 \quad = \text{DNE}$$

y': DNE $+$ 0 $-$ DNE ; x: -5, -2, 1

CVS: $x=-2,-5,1$

evs: $y=3,0,0$ $\therefore$ $(-2,3),(-5,0),(1,0)$

range: $y\in[0,3]$



7. $y=\sqrt{x^3+3x^2-6x-8}$ on $x\in[-4,4]$

zeros: $(-4,0),(-1,0),(2,0)$ ← from calc

domain: $x\in[-4,-1]\cup[2,4]$ $\quad y^2$: $-$ END $+$ 0 $-$ 0 $+$ END ; x: -4, -1, 2, 4

y-int: none b/c $\sqrt{-8}=i$

VA: none

POE: none

EB: none b/c domain

extremes: $y'=\frac{1}{2}(x^3+3x^2-6x-8)^{-1/2}(3x^2+6x-6)$

$$=\frac{3(x^2+2x-2)}{2(x^3+3x^2-6x-8)^{1/2}}=0 \leftarrow \text{quad. form} \quad = \text{DNE}$$

y': END DNE $+$ 0 $-$ DNE DNE $+$ END ; x: -4, -2.732, -1, 2, 4

CVS: $x=-2.732,\ \cancel{0.732},\ -4,-1,2,4$

evs: $y=3.224$, DNE, $0,0,0,8.994$ $\therefore$ $(-2.732,3.224),(-4,0),(-1,0),(2,0),(4,8.944)$

range: $y\in[0,8.994]$

8. $y = -\sqrt{6x - x^2 - x^3}$

zeros: $-x(x^2+x-6)=0$

$-x(x+3)(x-2)=0$

$(0,0), (-3,0), (2,0)$

domain: $x \in (-\infty, -3] \cup [0, 2]$

y^2: $+$ (0) $-$ (0) $+$ (0) $-$

x: -3, 0, 2

VA: none

POE: none

EB: none on right, ↓ on left

$\lim_{x \to -\infty} -\sqrt{-\infty - \infty^2 + \infty^3} = -\sqrt{\infty}$

y-int: $(0,0)$

extremes: $y' = -\frac{1}{2}(6x - x^2 - x^3)^{-1/2}(6 - 2x - 3x^2)$

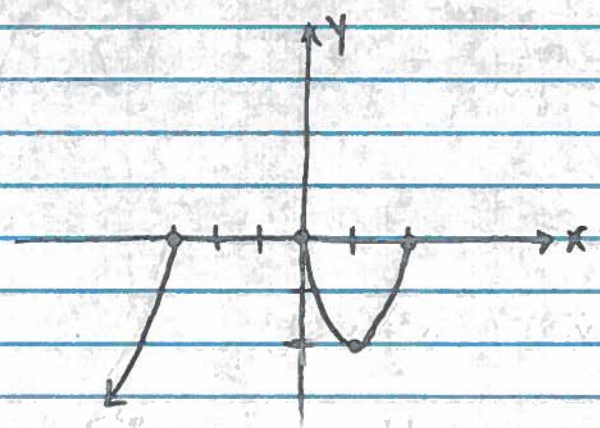
$= \dfrac{3x^2 + 2x - 6 = 0}{2(6x - x^2 - x^3)^{1/2} = \text{DNE}}$

CVS: $x = \cancel{-1.786}, 1.120, -3, 0, 2$

y': $+$ DNE -3 DNE $-$ 0 $+$ 1.120 DNE 2

RVS: $y =$ DNE, $-2.015, 0, 0, 0$

$\therefore (-3,0), (0,0), (1.120, -2.015), (2,0)$



9. $y = -\sqrt{\dfrac{x^2-1}{x^2-4}}$

zeros: $x^2-1=0$

$(1,0), (-1,0)$

domain: $x \in (-\infty,-2) \cup [-1,1] \cup (2,\infty)$

y^2: + DNE − 0 + 0 − DNE +

x: −2, −1, 1, 2

y-int: $\left(0, -\frac{1}{2}\right)$

VA: $x^2-4=0$

$x = \pm 2$

POE: none

EB: HA @ $y = -\sqrt{\frac{1}{1}}$ ∴ $y=-1$ on left + right

extremes: $y' = \dfrac{-1}{2}\left(\dfrac{x^2-1}{x^2-4}\right)^{-1/2} \cdot \dfrac{(x^2-4)(2x)-(x^2-1)(2x)}{(x^2-4)^2}$

$= \dfrac{-(x^2-4)^{1/2} \cdot \cancel{2}x(\cancel{x^2}-4-\cancel{x^2}+1)}{\cancel{2}(x^2-1)^{1/2}(x^2-4)^2}$

$= \dfrac{-3x(x^2-4)^{1/2}}{(x^2-1)^{1/2}(x^2-4)^2}$

$y' = \dfrac{-3x}{(x^2-1)^{1/2}(x^2-4)^{3/2}} \begin{matrix} =0 \\ = \text{DNE} \end{matrix}$

cvs: $x = 0, -1, 1, \cancel{-2}, \cancel{2}$

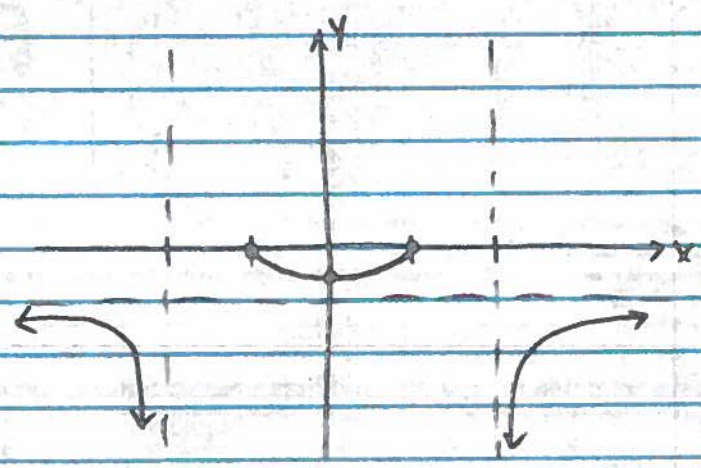
evs: $y = \frac{-1}{2}, 0, 0, \text{DNE}, \text{DNE}$

y': − DNE DNE − 0 + DNE DNE +

x: −2, −1, 0, 1, 2

∴ $\left(0, -\frac{1}{2}\right), (-1,0), (1,0)$

range: $y \in (-\infty,-1) \cup \left[-\frac{1}{2}, 0\right]$



10. $y=\sqrt{\dfrac{2x^2+7x-4}{x^3+3x^2-6x-8}}=\sqrt{\dfrac{(2x-1)\cancel{(x+4)}}{\cancel{(x+4)}(x+1)(x-2)}}=\sqrt{\dfrac{2x-1}{x^2-x-2}}$

$x=\frac{1}{2},-4,-1,2$

y^2: $-$ DNE $-$ DNE $+$ 0 $-$ DNE $+$

x: -4, -1, $\frac{1}{2}$, 2

Domain: $x\in\left(-1,\frac{1}{2}\right]\cup(2,\infty)$

zeros: $\left(\frac{1}{2},0\right)$

VAs: $x=-1$, $x=2$

POEs: none (b/c $x=-4$ is not in domain)

EB: none on left

HA $y=0$ on right

y-int: $\left(0,\frac{1}{\sqrt{2}}\right)$ or $(0,.707)$

extreme points: $\left(\frac{1}{2},0\right)$

Range: $y\in[0,\infty)$

$$y'=\frac{1}{2}\left(\frac{2x-1}{x^2-x-2}\right)^{-1/2}\cdot\frac{2(x^2-x-2)-(2x-1)(2x-1)}{(x^2-x-2)^2}$$

$$=\frac{1}{2}\cdot\frac{(x^2-x-2)^{1/2}}{(2x-1)^{1/2}}\cdot\frac{2x^2-2x-4-4x^2+4x-1}{(x^2-x-2)^2}$$

$$=\frac{-2x^2+2x-5}{2(2x-1)^{1/2}(x^2-x-2)^{3/2}}$$

i) $-2x^2+2x-5=0$

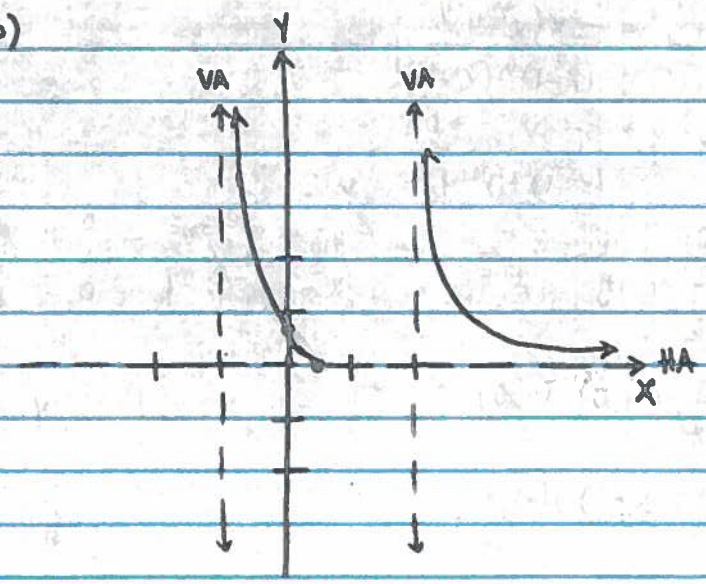
$x=\frac{1}{2}\pm\frac{3}{2}i$ ∴ none

ii) DNE @ $x=\frac{1}{2},\cancel{-1},\cancel{2}$ (not in domain)

CV: $x=\frac{1}{2}$, CV: $y=0$

y': DNE $-$ 0 DNE $-$

x: -1, $\frac{1}{2}$, 2



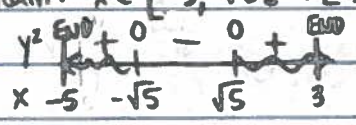
11. $y = -\sqrt{x^4 - 3x^2 - 10}$ on $x \in [-5, 3]$

zeros: $x^4 - 3x^2 - 10 = 0$

$(x^2 - 5)(x^2 + 2) = 0$

$(\pm\sqrt{5}, 0) \approx (\pm 2.236, 0)$

domain: $x \in [-5, -\sqrt{5}] \cup [\sqrt{5}, 3]$



y-int: none

POE: none

VA: none

EB: $\lim_{x \to \pm\infty} -\sqrt{\infty^4}$, so ↓ on both ends

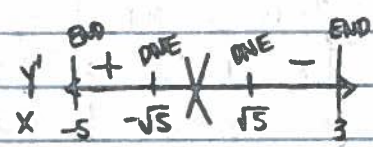
extremes: $y' = -\frac{1}{2}(x^4 - 3x^2 - 10)^{-1/2}(4x^3 - 6x)$

$= \frac{-\cancel{2}x(2x^2 - 3)}{\cancel{2}(x^4 - 3x^2 - 10)^{1/2}} = 0$ / DNE

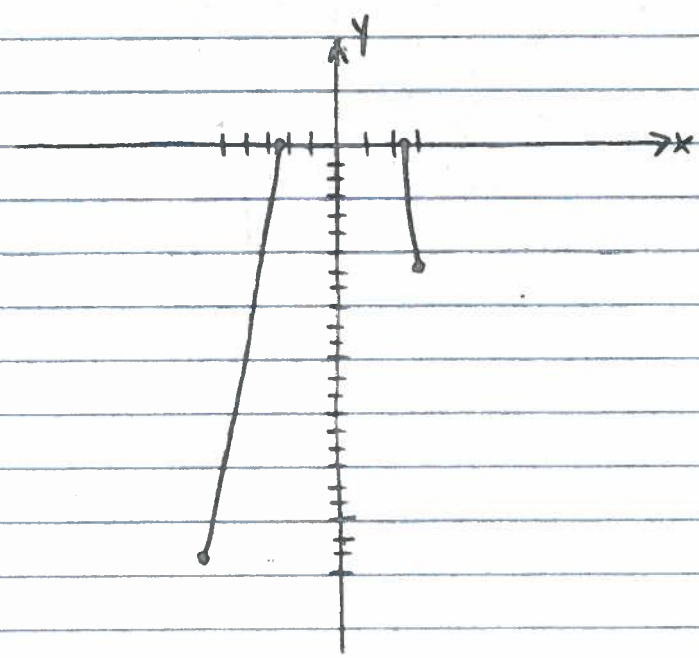
CVs: $x = \cancel{0}, \cancel{1.225}, \cancel{-1.225}, -\sqrt{5}, \sqrt{5}, -5, 3$

evs: $y =$ DNE, DNE, DNE, 0, 0, −23.238, −6.633

$\therefore (\pm\sqrt{5}, 0), (-5, -23.238), (3, -6.633)$



range: $y \in [-23.238, 0]$



12. $y=\sqrt{-x^4+13x^2-36}$

zeros: $x^4-13x^2+36=0$

$(x^2-9)(x^2-4)=0$

$(\pm 3,0),(\pm 2,0)$

domain: $x\in[-3,-2]\cup[2,3]$

y^2: $-$ 0 $+$ 0 $-$ 0 $+$ 0 $-$

x: -3 -2 2 3

y-int: none

VA: none

POE: none

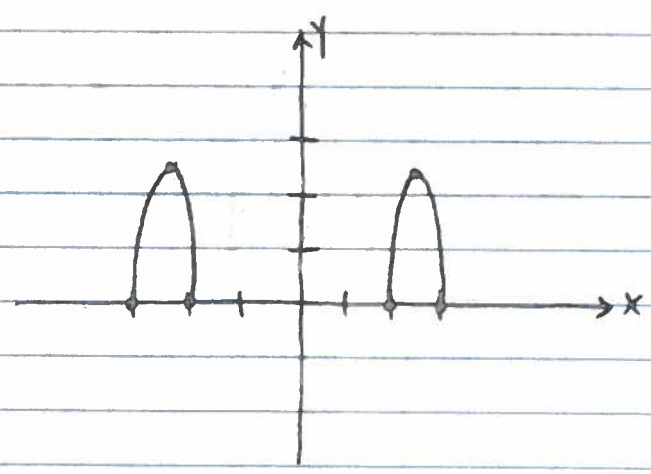
EB: none

extremes: $y'=\frac{1}{2}(-x^4+13x^2-36)^{-1/2}(-4x^3+26x)$

$=\frac{-\cancel{2}x(2x^2-13)}{\cancel{2}(-x^4+13x^2-36)^{1/2}}=0$ $=$ DNE

cvs: $x=\cancel{0},\ \pm 3,\ \pm 2,\ \pm 2.550$

cvs: $y=$ DNE, 0, 0, 2.5

$\therefore (\pm 3,0),(\pm 2,0),(\pm 2.550,\ 2.5)$

range: $y\in[0,2.5]$